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AN ANALYSIS OF THREE-DIMENSIONAL NON-PLANAR CRACK PROPAGATION PHENOMENON WITH SMOOTHED PARTICLE HYDRODYNAMICS METHOD

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Abstract. Abstract. In the present study, the non-planar crack propagation problems in the 3D body are solved, extending our previous study on the smoothed particle hydrodynamics (SPH) method applied to the fatigue crack propagation of the planar cracks in the 3D body. To solve the propagation of the non-planar crack, the crack front particles are given the information of the slope and the position of the crack surface in addition to the crack length. To confirm the validity of the proposed method, a fatigue test of the CT specimen with an additional horizontal hole is carried out and the result is compared with the computed one successfully.

1 INTRODUCTION

The fatigue damage is the critical problem for almost all the mechanical structures. Fatigue crack propagation is one of the dominant phenomena of the damage. Accordingly, some meshing numerical analyses for the crack propagation problem, represented by X-FEM [1], have been investigated. However, in general, it is difficult to solve some complex situations, such as the process of penetrating the thickness of a plate, or connecting multiple cracks or defects [2,3]. On the other hand, mesh-less numerical analyzes, like a particle method, is expected to solve such problems easily.

In the preceding study, we applied the smoothed particle hydrodynamics (SPH) method [4] to the linear fracture mechanics and the fatigue crack propagation and proposed the method to solve the propagation of the planar crack in the 3D body with such complex situations [5].

In the present study, the method to solve the propagation of the non-planar cracks in the 3D body is proposed. In addition, to confirm the validity of the method, a fatigue test of the non-planar crack is carried out and the result is compared with the computed result.

2 ANALYSIS METHOD

2.1 Basic algorithm for fatigue crack propagation

In the present method, the basic algorithm for the crack propagation is the same as the method for the planar cracks [5]. The method consists of two sections, such as the SPH stress analysis section and the fatigue crack propagation analysis section. At first, the stress distribution of the 3D body is computed in the SPH section, and the result is sent to the crack propagation analysis. The support domain size in the SPH is twice the particle size.

In the crack propagation analysis, the values of the stress intensity factor of the crack front particle i, K_i, is calcluated simply as follows:

$$K_{\mathrm{i}} = \sigma_{1,\mathrm{i}}(\pi\Phi)^{1/2} \tag{1}$$

where $\sigma_{1,\,i}$ is the maximum principal stress and Φ is the size of the particle. In the n^{th} analysis step, the fatigue cycle increment ΔN_n is given and the increment of the crack length in the particle $L_{i,n}$ is calculated as follows:

$$\Delta L_{i,n} = \mathrm{d}a/\mathrm{d}N|_i \, \Delta N_n \tag{2}$$

where $\mathrm{d}a/\mathrm{d}N|_i$ is the fatigue crack growth rate of the particle. It is obtained by equation (1), given stress ratio R and Paris Erdgan Law [6]. The crack length of the particle in this step $L_{i,n}$ is calculated as follows:

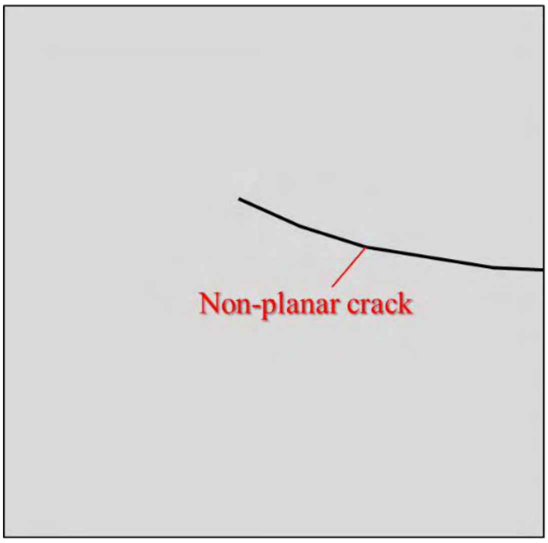
$$L_{i,n} = L_{i,n-1} + \Delta N_n \tag{3}$$

where $L_{i,n-1}$ is the crack length of the particle in $(n\text{-}1)^{\mathrm{th}}$ analysis step. When $L_{i,n}$ becomes the same value as Φ, the particle becomes the fractured particle. The decreasing of the stiffness due to the crack propagation is realesed by removing the fractured particle.

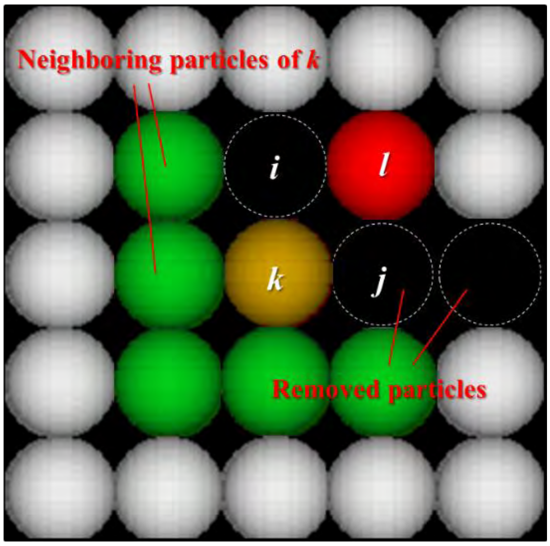
2.2 Non-planar crack modeled by particles

Fig. 1 shows the example of the non-planar crack in the 3D body. The non-planar crack in Fig. 1 (a) is modeled by the particles as shown in Fig. 1(b). In the figure, the particles *i* and *j* are already removed due to the crack propagation and the particles *k* and *l* are placed on the uppser side and the lower side of the cracked surface, respectively. Before the particles *i* and *j* are fractured, the particles *k* and *l* were the neighboring particle each other. However, when both of the particles *i* and *j* are fractured, the crack appears between the particles *k* and *l*, then, the particles should not be the neighboring particle in the figure.

To realize the above state, the particles on the same position as the particle *l* in the figure are searched in the nine cross sections, A to I in Fig. 2, of all the groups of the neiboring particles. As shown in Fig. 3, the neighboring particles of the particle *k* are numbered by the relative positions between the particle *k*. In case of the cross section A in Fig. 2, the particles shown in Fig.4 are searched. In case of the figure, the particles No. 14 and 16 are the removed particles and the particle No. 17 is the same state as the particle *l* in Fig. 1(b). Then, in the figure, the particle No. 17 is excluded from the group of neighboring particles of the particle *k*.



(a) Schematic view in a cross section



(b) Particle model of (a)

Figure 1: Non-planar crack in 3D body

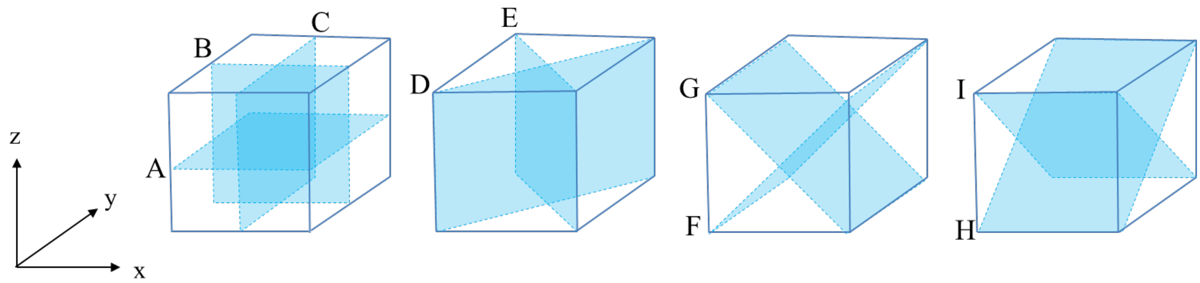


Figure 2: Searching planes in group of neighboring particles

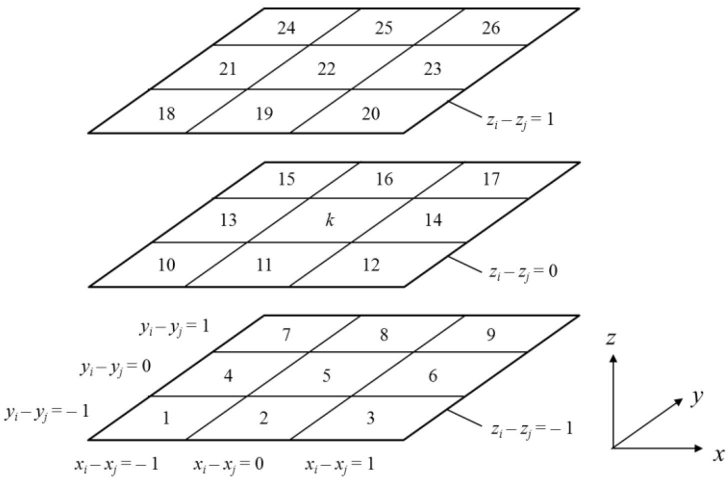


Figure 3: Numbers for neighboring particles

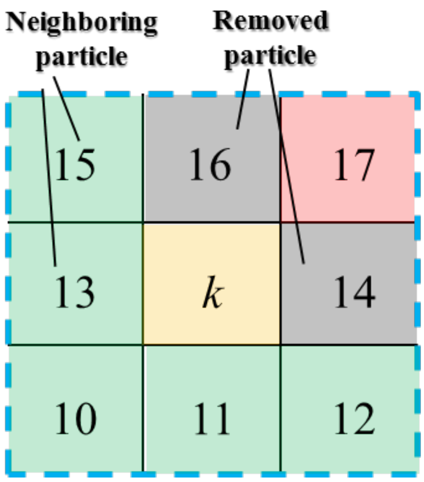


Figure 4: Plane A in Fig .2

2.3 Crack propagation algorithm for non-planar crack

To calculate the propagation of the non-planar crack, the slope and the position of the crack surfaces in the crack front particles should be concerned. Then, in the present method, the crack front particles have information of the slope and the position of the crack surface as shown in Fig. 5 in addition to the crack length. The axis of ζ in the figure is decided as the same direction of x, y or z which is most close to the direction of σ_1 in the initial crack front. Thus, ξ-η-ζ is x-y-z, y-z-x or z-x-y. As shown in the left side of the figure, the information of the slope is given as θ_ξ and θ_η. They are the angles between the orthogonal plane of σ_1 and the axis of ξ and η, respectively. As shown in the right side of the figure, the position is given as the distance from bottom ξ-η plane. The new crack front particle receives h_1, h_2, h_3 or h_4 of the fractured particle as its h_3, h_4, h_1 or h_2 and the other values are calculated by θ_ξ and θ_η.

When the crack front particle is fractured, the new crack front particles are searched in the sub groups of the neighboring particles I to IV in Fig. 6. For example, in case of Fig. 7, the sub group II consists of the neighboring particles No. 6, 14 and 23 (see Fig. 3). If the group does not consist of any other crack front particle nor removed particle when the particle i is fractured, one of these will be defined as a new crack front particle. If h_2 of the particle i is larger than 0 and smaller than Φ, the particle No. 14 will be defined as the new crack front particle and it recives h_2 of the particle i as its h_4. If h_2 of the particle i is smaller than 0,

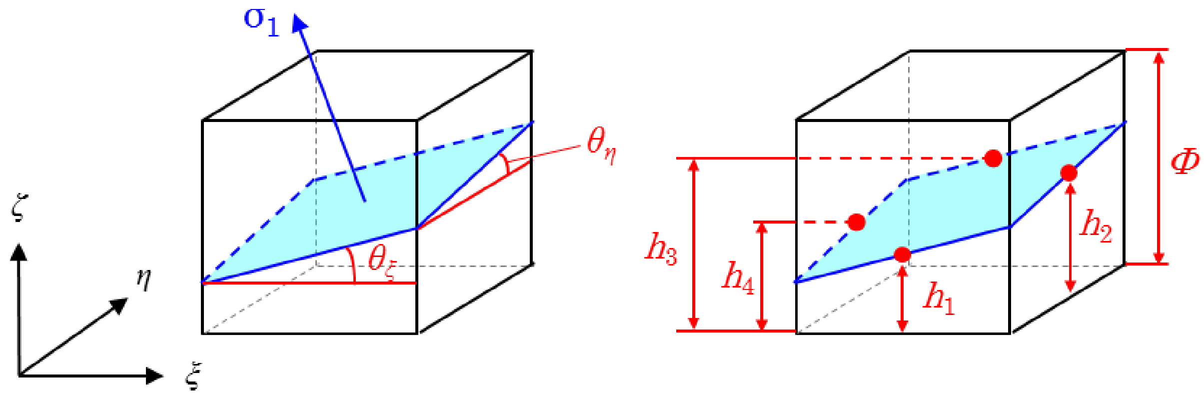


Figure 5: Schematics of slope and position of crack surface in crack front particle

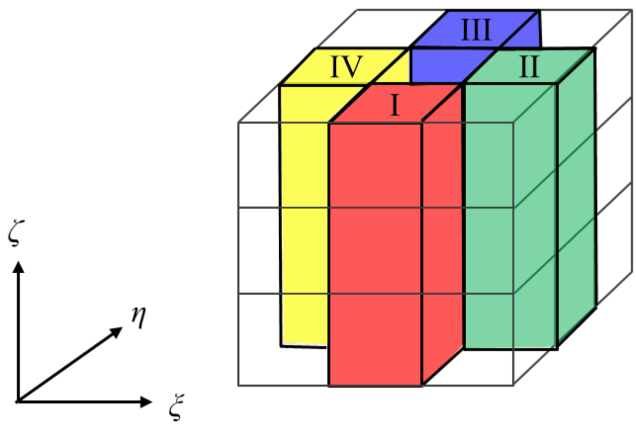


Figure 6: Groups of neighboring particles

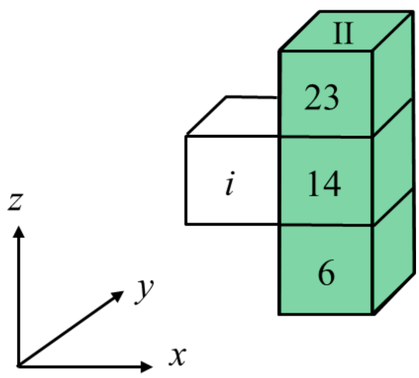


Figure 7: Example of group II in Fig. 6

the particle No. 6 will be defined as the new crack front particle and it recives the sum of h_2 of the particle i and Φ as its h_4. If h_2 of the particle i is larger than Φ, the particle No. 23 will be defined as the new crack front particle and it recives the difference of h_2 of the particle i and Φ as its h_4. Using the above method, the non-planar crack propagation is analyzed.

3 FATIGE TEST

To confirm the validity of the method, a fatigue crack propagation test was carried out. Fig. 8 shows the giometory of the fatigue specimen. The specimen is the half inch CT specimen [7] with an additional horizontal hole. The hole is 10mm in diamiter and places 8mm bellow from the center line and 25 mm from the back surface of the specimen. The specimen is made of ductile cast iron. The tensile cyclic load was applied to the specimen by an electro-hydraulic servo fatigue testing machine.

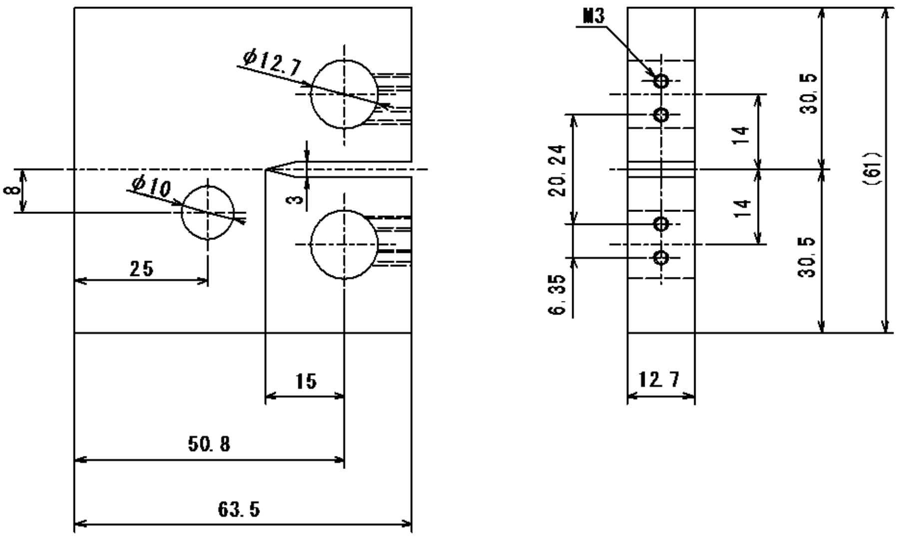
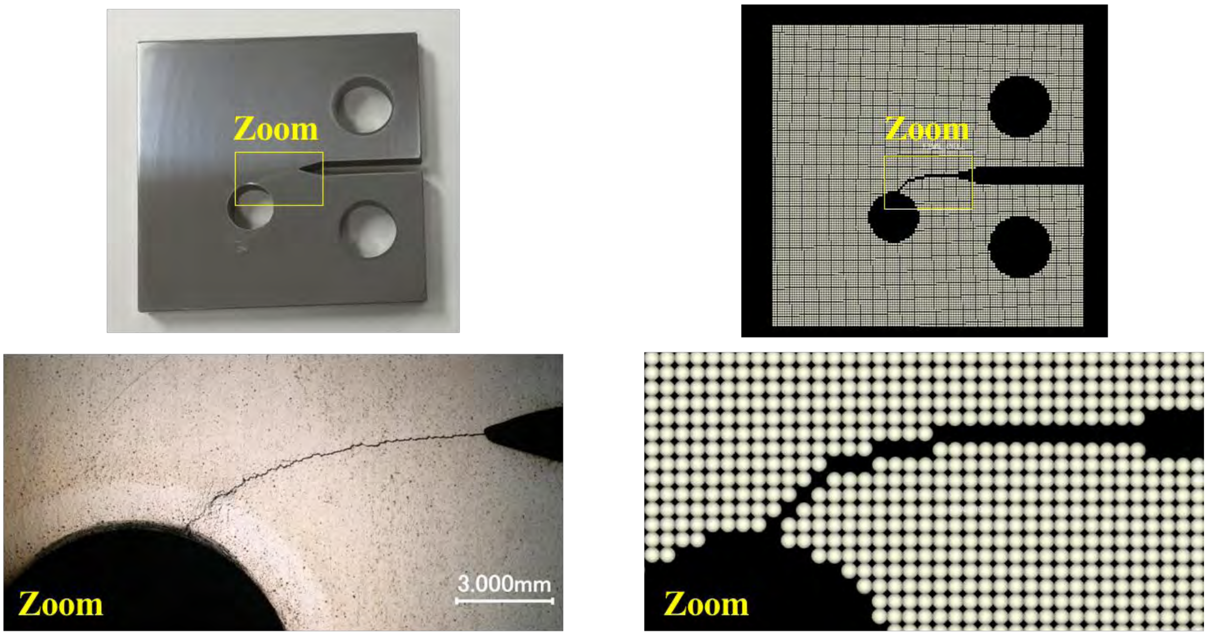


Figure 8: CT Specimen with an additional horizontal hole

4 COMPUTATIONAL AND TESTED RESULTS

Fig. 9 shows the tested and the computed results of the crack propagation of the CT specimen with an additional horizontal hole. The computed specimen consists of 70, 000 particles and they are placed at regular intervals, meaning that the diameter of a particle is 0.5 mm.

As shown in Fig. 9 (a), the tested result showed that the initial crack propagated to horizontal direction and the crack changed its direction to the additional hole step by step. Finally, the crack passed though the edge of the hole. As shown in Fig. 9 (b), the computed result shows the smoothed crack propagation and the result shows almost the same crack shape as the tested result.



(a) Tested result (b) Computational result

Figure 9: Crack propagation of CT specimen with an additional horizontal hole

5 CONCLUSIONS

- The method for the 3D planar crack propagation by SPH is extended to solve the 3D non-planar crack propagation.
- The non-planar crack propagation of CT specimen with an additional horizontal hole is computed in a smooth manner.
- The computed result shows almost the same crack shape as the tested result.

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